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ABSTRACT

The goal of this pamphlet is to provide instructors of various scientific disciplines with mathematically accurate graphs of elementary polynomial functions. The figures in this pamphlet are intended to provide suitable material for the preparation of classroom handouts and overhead transparencies. In addition, sample sets of exercises are provided for each figure. Grid lines are provided on all graphs for convenience in working with problems related to translation of axes. Each graph has the same scaling on the X and Y axes to make it easier to interpret the slope of any curve. (MP)

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MATHEMATICS

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Volume 1: Polynomials

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PREFACE

The goal of this pamphlet is to provide instructors of various scientific disciplines with mathematically accurate graphs of elementary polynomial functions. The figures in this pamphlet are intended to provide suitable material for the preparation of classroom handouts and overhead transparencies.

In addition, sample sets of exercises are provided for each figure. The physical size of each figure is 10 cm. by 10 cm. so that coordinate information can be easily measured with a metric ruler. Each graph has the same scaling on the x and y axes to make it easier to interpret the slope of any curve. Grid lines are provided on all graphs for convenience in working with problems related to translation of axes.

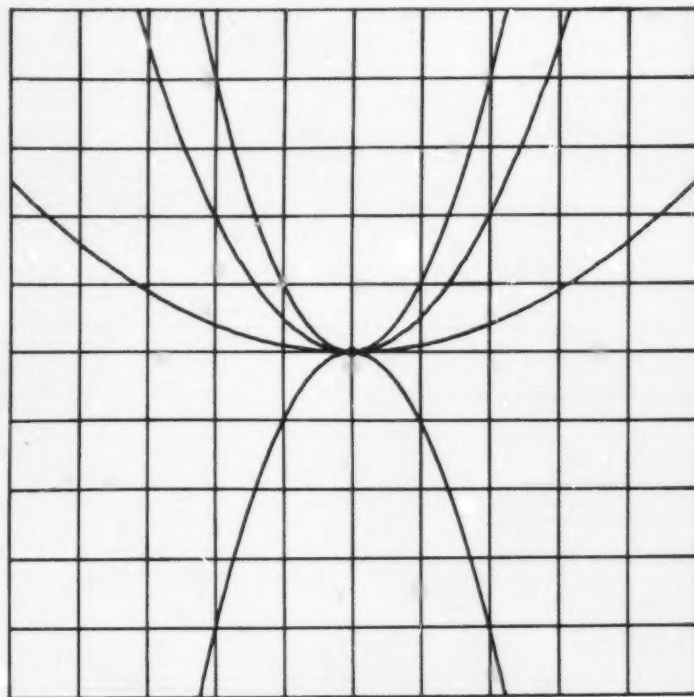
If students are to learn accuracy with respect to formulating hypotheses and constructing proofs they should also be familiar with graphs which are accurate. A great deal can be learned by studying accurate representations of mathematical curves.

This pamphlet was a joint project of the Mathematics and Computer Sciences Department and Computer Services at Eastern Washington University. The material was prepared by Herbert L. Holden. Design and lettering was done in the Graphics Lab of the Instructional Media Center.

$$y = ax^2 \quad a = -1, 0.1, 0.5, 1$$

1

Scale: -5 to 5

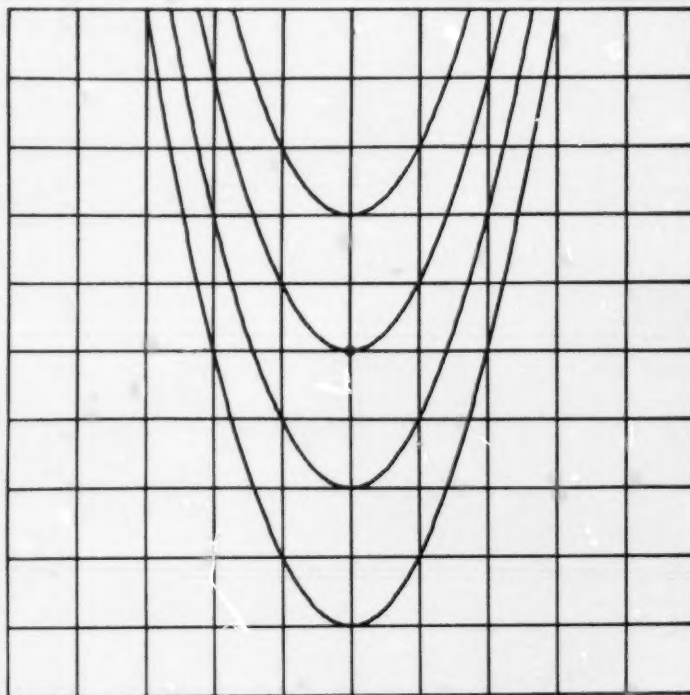


1. Which curve passes through the point $(-2, 2)$?
Which one passes through the point $(2, 4)$?
2. Which curve has the greatest slope when $x = 2$?
3. Using a straight-edge estimate the slope of each curve at $x = -1$ and $x = 2$.
4. From the graph estimate the value of x^2 for $x = 1.5$ and $x = 0.5$.
5. From the graph of $y = x^2$ estimate $\sqrt{3}$ and $\sqrt{5}$.
6. From the graph of $y = 0.1x^2$ estimate $\sqrt{10}$.

2

$$y = x^2 + a \quad a = -4, -2, 0, 2$$

Scale: -5 to 5

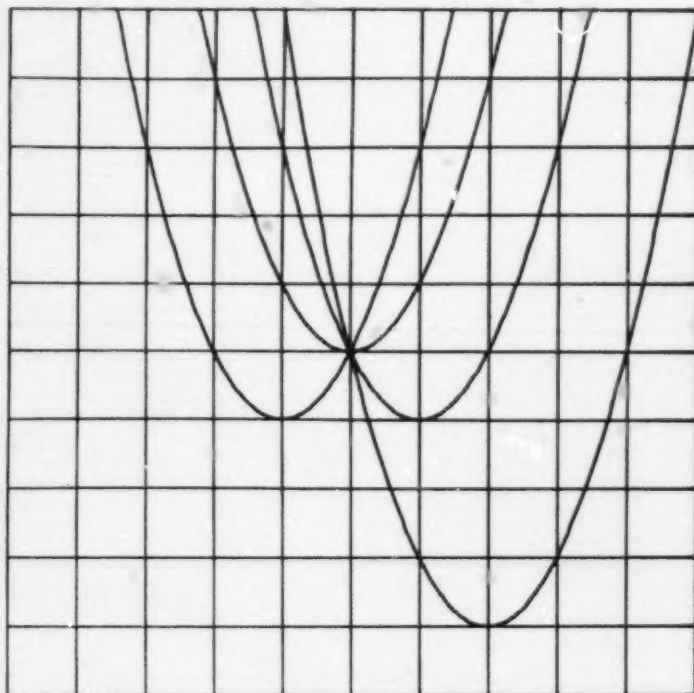


1. Estimate where each of these curves cross the x -axis.
2. Using a straight edge estimate the slope of each of these curves at $x = -1, 0$, and $+1$.
3. Estimate the area between curve $y = x^2 - 4$ and the x -axis.
4. Define the "translate" of a curve.
5. Can you determine the roots of $y = x^2 - 3$ from this figure?
6. If $y = x^2 + a$ passes through the point $(0, 3)$ what is " a "?

$$y = x^2 + ax \quad a = -4, -2, 0, 2$$

3

Scale: -5 to 5

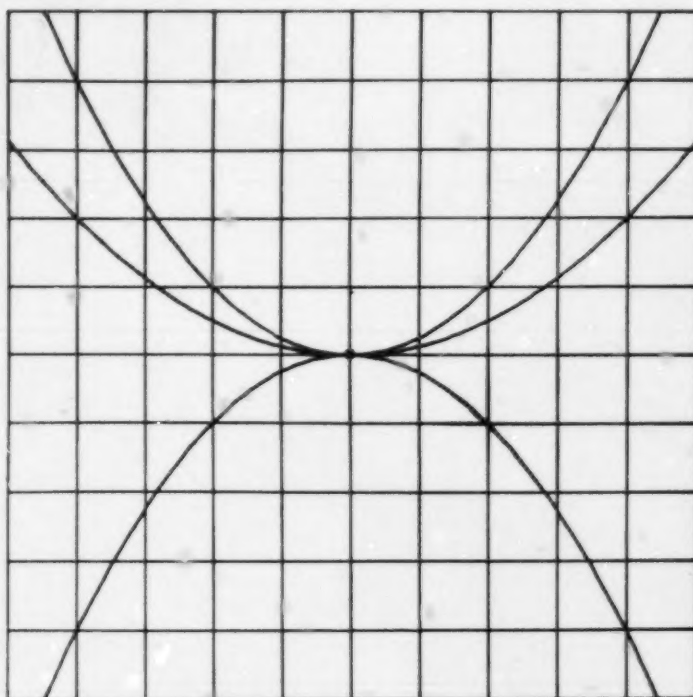


1. Estimate the slope of each of these curves at $x=1$ and $x=2$.
2. Estimate the roots of each of these curves.
3. Estimate the roots of $y = x^2 - 4x + 2$.
4. Does the equation $y = x^2 + ax$ always pass through the origin?
5. Show that $y = x^2 + 2x$ is a translate of $y = x^2 - 2x$.
6. Show that $y = x^2 + ax$ is a translate of $y = x^2$.

4

$$x^2 = 4ay \quad a = -1, 1, 2$$

Scale: -5 to 5

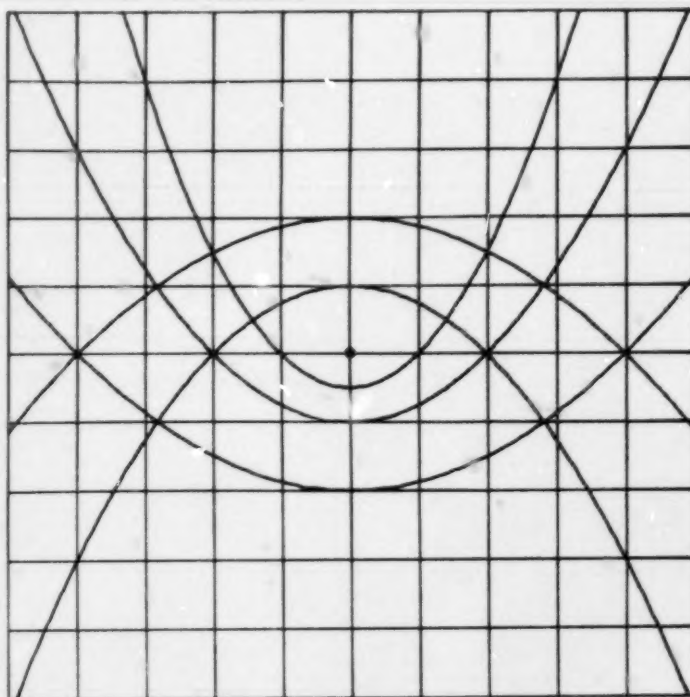


1. What is the focus and directrix for each parabola?
2. A parabola is the locus of points which are equidistant from a focus and directrix. Verify this by direct measurement.
3. Define symmetric with respect to the y-axis and symmetric with respect to the x-axis.
4. What sort of symmetry does the curve $x^2 = 4ay$ have?
5. Find the focus of $x^2 = -4y$ and by use of a straight-edge and protractor determine how light rays emanating from the focus are reflected by the parabola.

$$y = \frac{x^2}{4a} - a \quad a = -2, -1, 0.5, 1, 2$$

5

Scale: -5 to 5

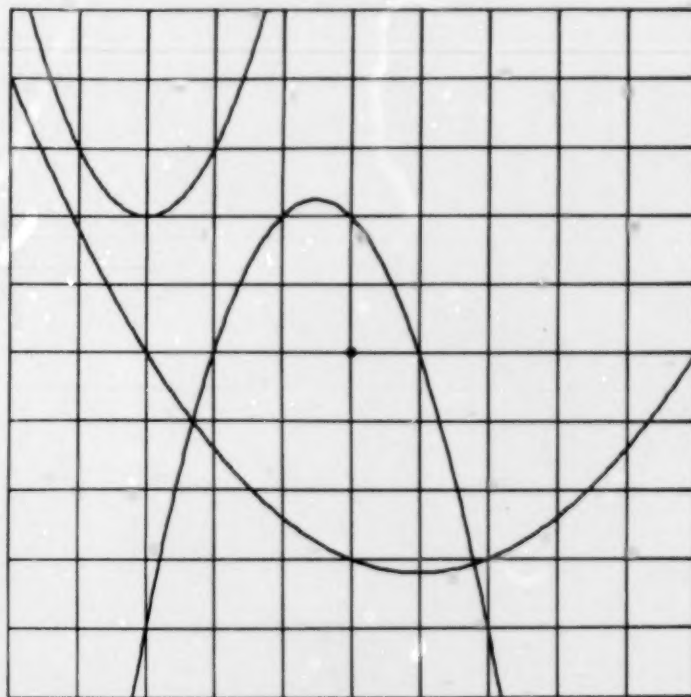


1. Estimate the intersection points for the two curves defined by $a = 2$ and $a = -1$.
2. By direct measurement, determine which intersections are perpendicular.
3. Find the focus for each of these curves.
4. Are all these curves symmetric with respect to the y -axis?
5. Define what it means for a piece of a curve to be "concave up" and "concave down".
6. For what values of " a " are these parabolas concave up?

6

$$y = -x^2 - x + 2$$

Scale: -5 to 5

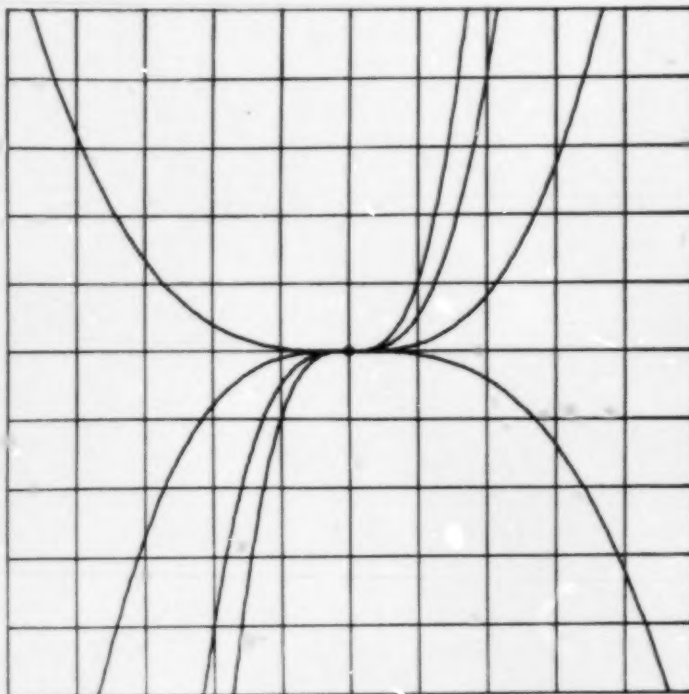


1. Determine the equation of the curve which passes through $(-2, 3)$.
2. Define "axis of symmetry" of a curve and estimate the axis of symmetry for each curve in this figure.
3. From the graph of $y = -x^2 - x + 2$ determine the roots of the functions $y = -x^2 - x + 3$ and $y = -x^2 - x + 6$.
4. Determine the equation of the curve which passes through $(0, -3)$.
5. Is one of these curves a translate of $y = x^2$?
6. Where does the function $y = -x^2 - x + 2$ attain its maximum value?
7. Does every second degree polynomial have a vertical axis of symmetry?

$$y = ax^3 \quad a = 1, 0.5, 0.1, -0.05$$

7

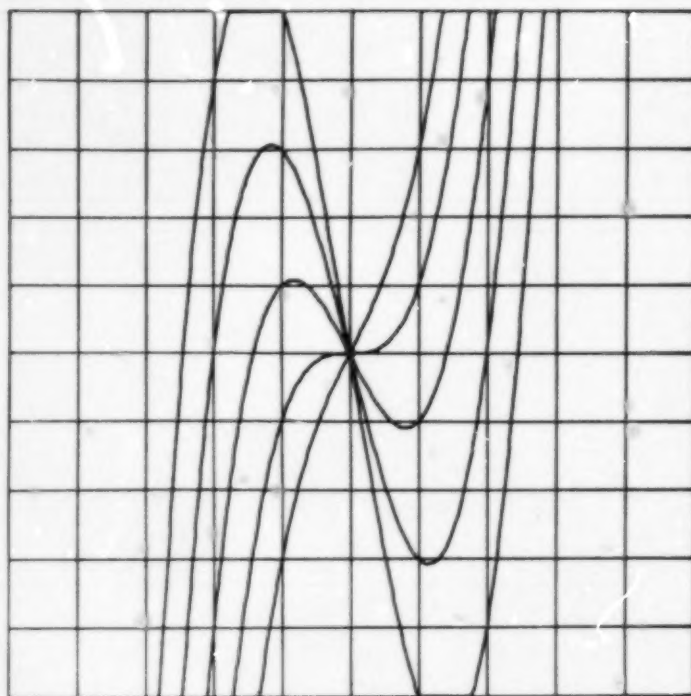
Scale: -5 to 5



1. Which curve passes through $(-3, -2.7)$?
2. Which curve has the greatest slope when $x = 1$?
3. Using a straight-edge estimate the slope of each curve at $x = 1$.
4. From the graph of $y = x^3$ estimate $\sqrt[3]{2}$, $\sqrt[3]{3}$, and $\sqrt[3]{4}$.
5. From the graph of $y = 0.5x^3$ estimate $\sqrt[3]{2}$.
6. Compare the slope of the curve $y = x^3$ with $y = x^2$ and discuss the differences and similarities.

8

$$y = x^3 + ax \quad a = 2, 0, -1, -3, -5$$



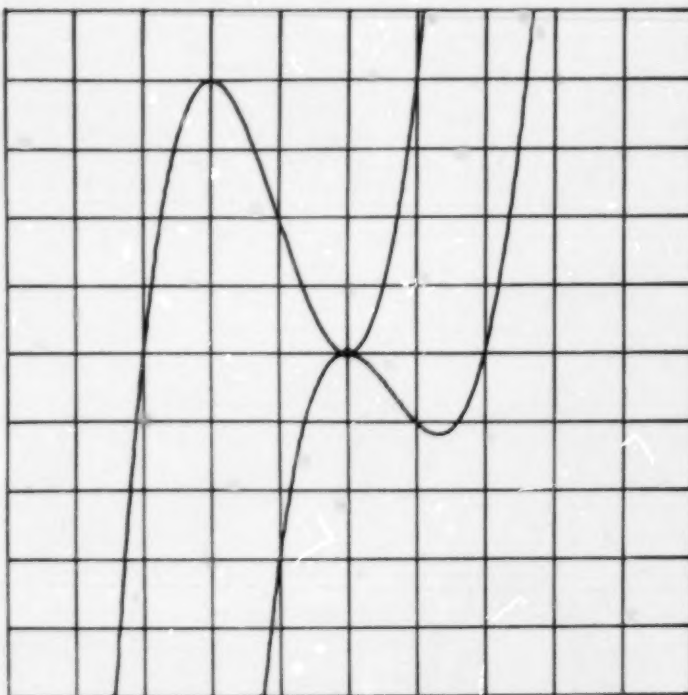
Scale: -5 to 5

1. Estimate the area between the curve $y = x^3$ and the lines $y = 0$ and $x = 1$.
2. Using a straight edge estimate the slope of each curve at $x = 1$.
3. Estimate the roots of each curve.
4. Is each of these curves symmetric with respect to the origin? Define "symmetric with respect to the origin".
5. How does the equation $y = x^3$ change if the origin is translated to $(1, 2)$?
6. Estimate the roots of $x^3 - 3x + 3$.

$$y = x^3 + 3x^2 \quad y = x^3 - 2x^2$$

9

Scale: -5 to 5

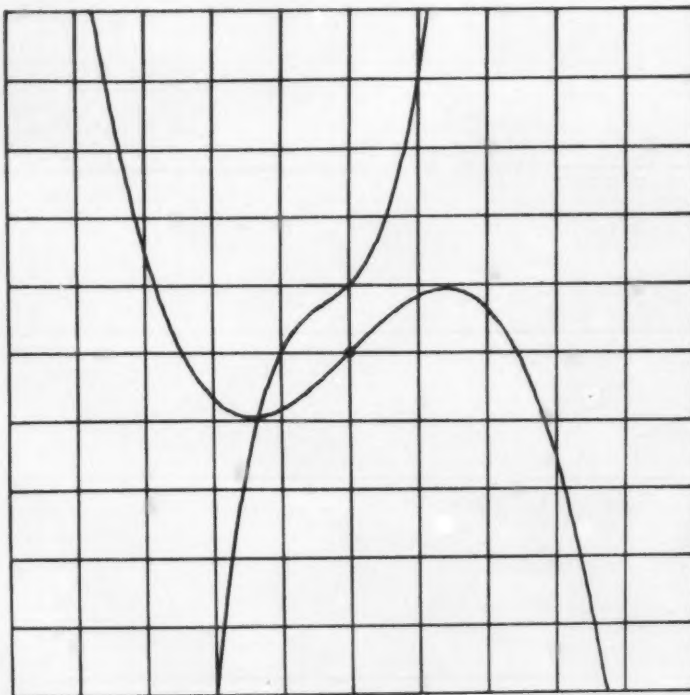


1. Define the turning points of a curve.
2. Define the local minimum, maximum and inflection point for a curve. Estimate these points for each curve.
3. Is every cubic symmetric with respect to its inflection point?
4. Estimate the roots of $x^3 + 3x^2 - 3$.
5. What happens to the general form of a cubic polynomial when the inflection point is translated to the origin?
6. Is one of these curves a translate of $y = x^3 - 3x$?

10

$$y = x^3 + x^2 + x + 1 \quad y = x - \frac{x^3}{6}$$

Scale: -5 to 5

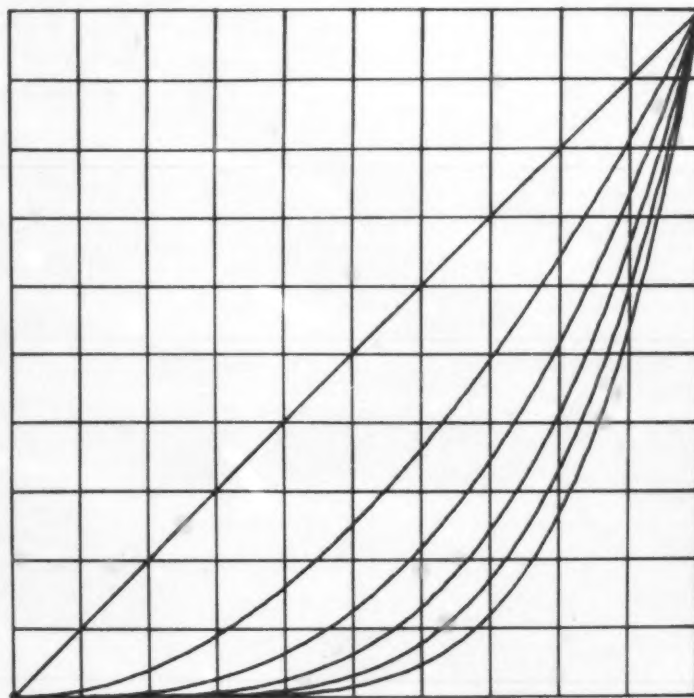


1. Find the inflection point for $y = x^3 + x^2 + x + 1$.
2. Compare the values of $y = x - \frac{x^3}{6}$ with $\sin x$ for $\pi/4 \leq x \leq \pi/4$ where x is measured in radians.
3. Will $y = x^3 + x^2 + x + a$ always have at least one real root?
4. Will every cubic polynomial have at least one real root? Is the same true for every quadratic polynomial?
5. Define the difference between a minimum value for a function and a local minimum.
6. Estimate any local minimums which appear in this figure.

$$y = x^n \quad n = 1, 2, 3, 4, 5, 6$$

11

Scale: 0 to 1

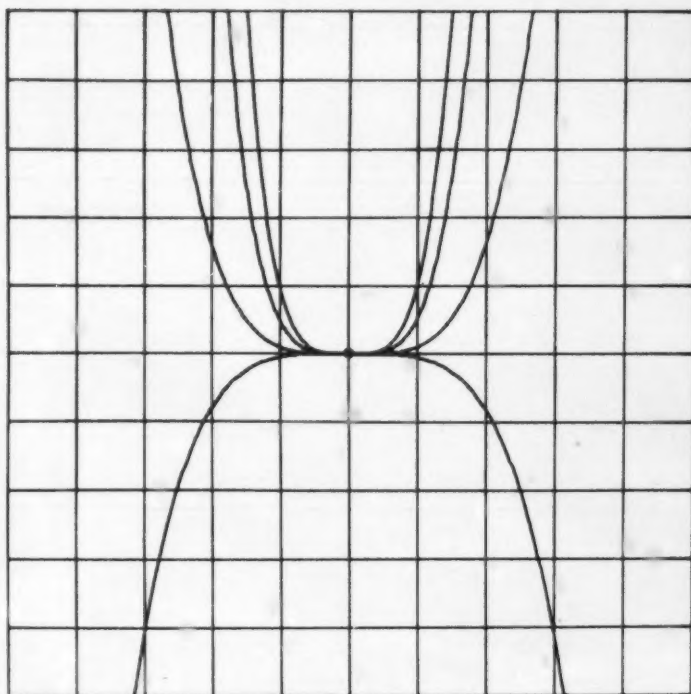


1. Estimate $\sqrt{2}$. What other roots can be estimated using this figure?
2. Discuss the behavior of the slope of x^n between $x=0$ and $x=1$.
3. When is the slope of x^n equal to 1?
4. Guess the slope of $y=x^n$ at $x=0$ for $n=1, 2, 3, 4, 5, 6$.
5. Estimate the area between $y=x^6$ and the lines $y=0$, $x=0.9$, $x=1$ by using the formula for the area of a trapezoid.
6. Guess the slope of $y=x^n$ at $x=1$ for $n=1, 2, 3, 4, 5, 6$.

12

$$y = ax^4 \quad a = 1, 0.5, 0.1, -0.05$$

Scale: -5 to 5

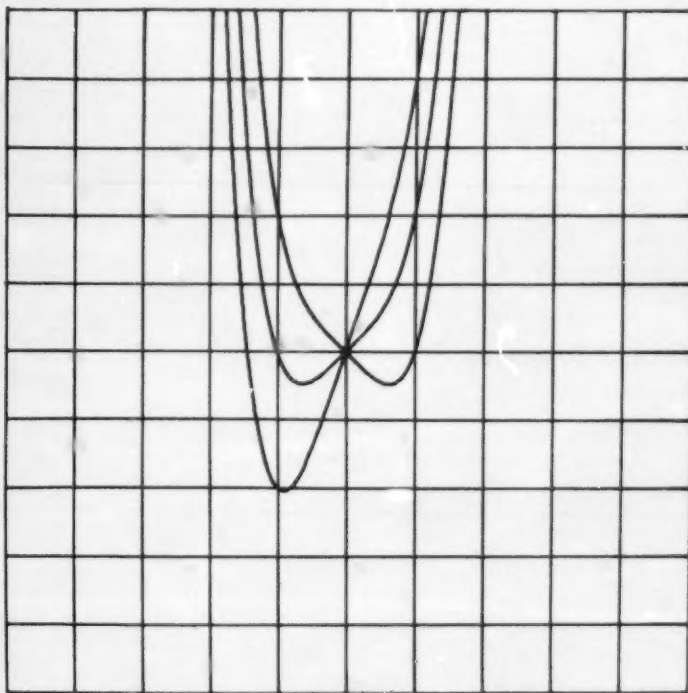


1. Compare this figure with figure 1 and discuss the differences. Do the same with figure 12 and figure 7.
2. Estimate the area between the curve $y = -0.05x^4$ and the line $y = -5$.
3. Which of these curves is the steepest at $x = 2$?
4. Estimate the area between $y = -0.05x^4$ and the lines $y = 0$, $x = 0$, and $x = 3$ by using the formula for the area of a trapezoid.
5. If $y = ax^4$ passes through the point $(5, 1)$, what is "a"?
6. Estimate the roots of $y = 0.1x^4 - 4$.

$$y = x^4 + ax \quad a = 1, -1, -3$$

13

Scale: -5 to 5

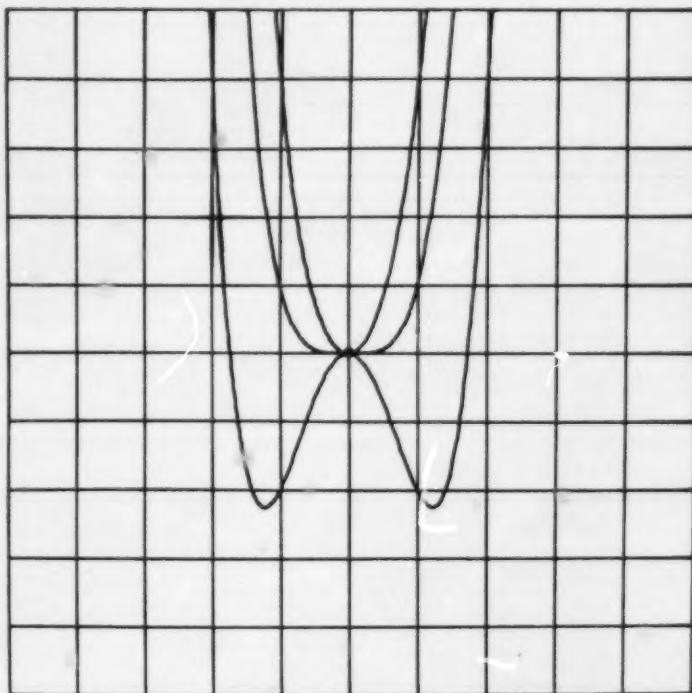


1. How many roots do each of these functions have and what are they?
2. Discuss the effect of "a" on the shape of $y = x^4 + ax$.
3. Will $y = x^4 + ax$ always pass through (0,0)?
4. Estimate the roots of $y = x^4 + x + 1$ and $y = x^4 + x - 1$.
5. Define "reflection about the y-axis".
6. Show that one of these curves can be obtained from another by reflection about the y-axis.

14

$$y = x^4 + ax^2 \quad a = 3, 0, -3$$

Scale: -5 to 5

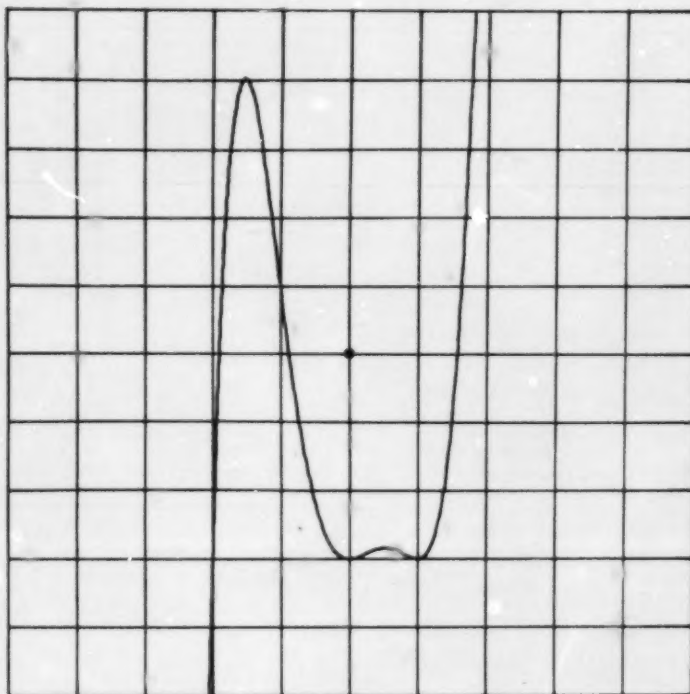


1. Discuss the effect of "a" on the shape of $y = x^4 + ax^2$.
2. Estimate the roots of $y = x^4 - 3x^2 + 1$.
3. What is the maximum number of roots that $y = x^4 + ax^2$ can have?
4. Must $y = x^4 + ax^2$ have at least one real root?
5. Must $y = x^4 + ax^2$ have at least one local minimum or maximum?
6. Discuss the last three questions for any fourth degree (quartic) polynomial.

$$y = x^5 - 3x^3 + 2x^2 - 3$$

15

Scale: -5 to 5

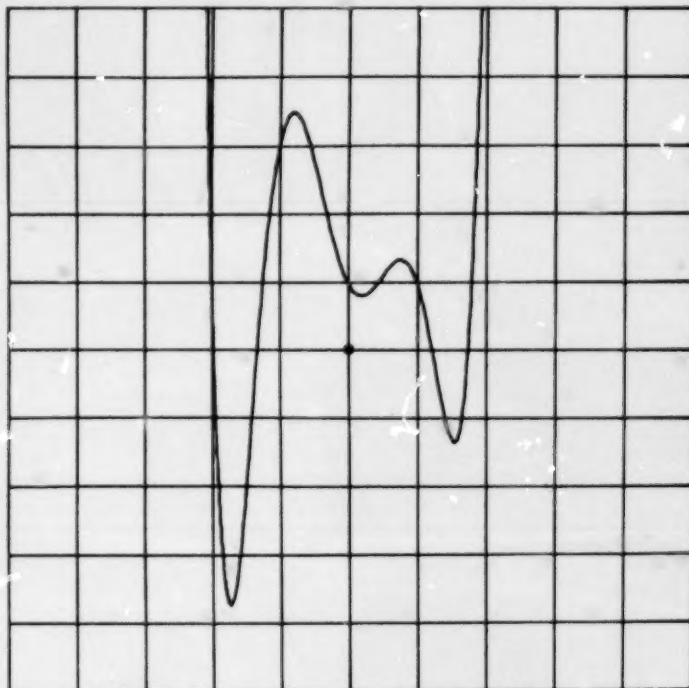


1. Estimate the turning points of this curve. How many are there?
2. Estimate the roots of $x^5 - 3x^3 + 2x^2$
3. Evaluate $y = x^5 - 3x^3 + 2x^2 - 3$ for $x = -5, -4, -3, 3, 4$, and 5 .
4. Do you think that a fifth degree polynomial must have at least one root?
5. How many local maximums does this function have?
6. How many local maximums can any fifth degree polynomial have?

16

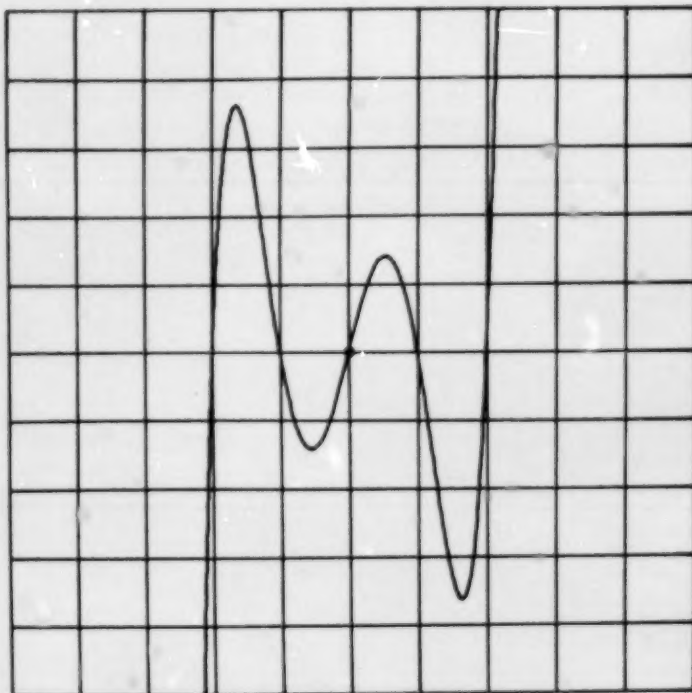
$$y = x^6 - 5x^4 + x^3 + 5x^2 - 2x + 1$$

Scale: -5 to 5



1. Estimate the turning points of this curve. How many are there?
2. Estimate the roots of this function. How many are there?
3. Estimate the roots of $y = x^6 - 5x^4 + x^3 + 5x^2 - 2x$.
4. Does a sixth degree polynomial always have an even number of real roots?
5. What can you say about this function for large positive values of "x"? What about large negative values of "x"?
6. What is the minimum value of this function for the following domains:
 - a. $0 \leq x \leq 1$
 - b. $0 \leq x \leq 2$
 - c. $-2 \leq x \leq 2$
 - d. $-1 \leq x \leq 0$

Scale: -5 to 5

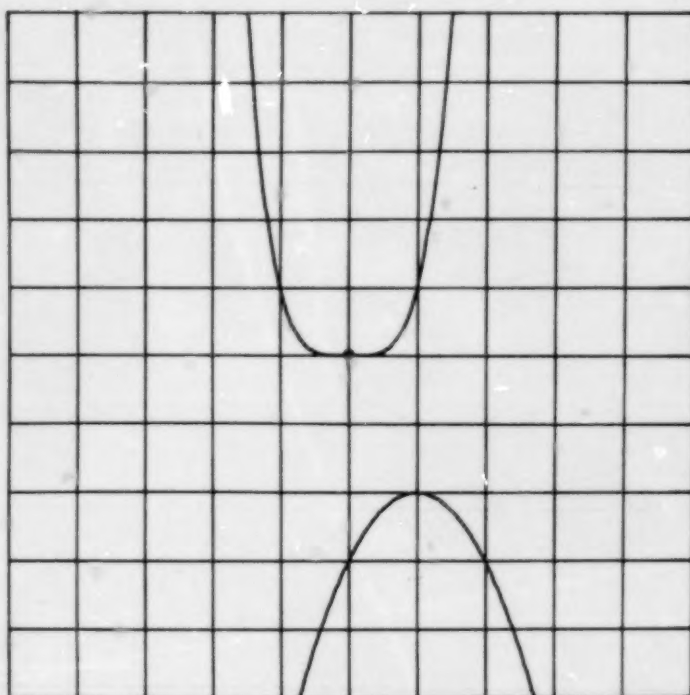


1. Guess the equation of this curve.

18

$$y = x^4 \quad y = -x^2 + 2x - 3$$

Scale: -5 to 5



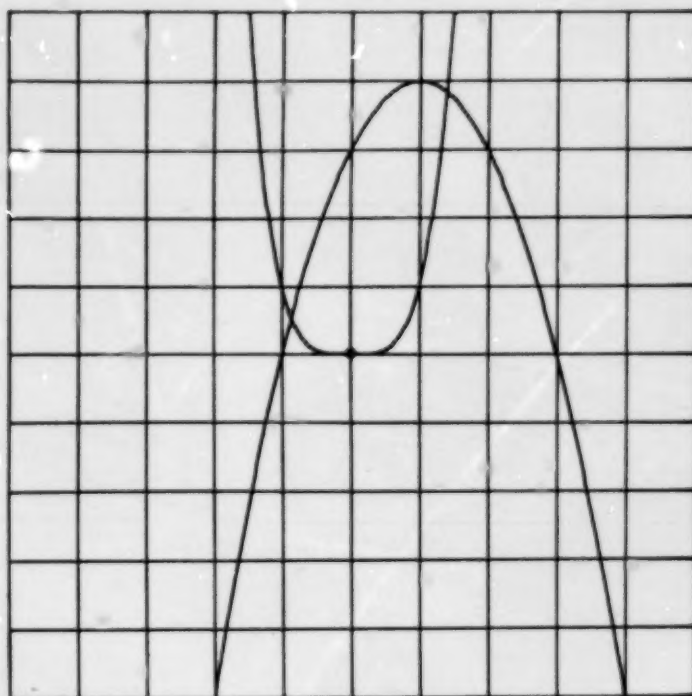
1. The x -coordinates of the points where the curves $y = x^4$ and $y = -x^2 + 2x - 3$ intersect are the values of " x " for which $x^4 = -x^2 + 2x - 3$ or $x^4 + x^2 - 2x + 3 = 0$.

Does $y = x^4 + x^2 - 2x + 3$ have any real roots?

$$y = x^4 \quad y = -x^2 + 2x + 3$$

19

Scale: -5 to 5

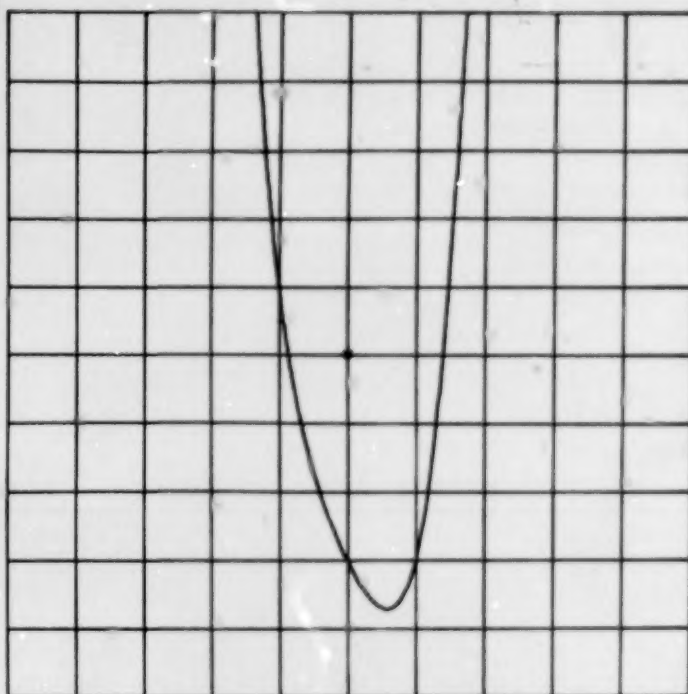


1. For fourth degree polynomials whose cubic term is missing, it is often easy to plot $y = x^4$ and a parabola. Compare this figure with figure 20. Try some examples of your own.
2. The equation $x^2 + ax + b = 0$ can be written as $x^2 = -ax - b$. Sketch $y = x^2$ and $y = -ax - b$ for various choices of "a" and "b".
3. $x^3 + ax^2 + bx + c = 0$ can be written as $x^2 + ax + b = -c/x$. Sketch $y = x^2 + ax + b$ and $y = c/x$ for various a, b, and c.

20

$$y = x^4 + x^2 - 2x - 3$$

Scale: -5 to 5



1. How many roots does $y = x^4 + x^2 - 2x - 3$ have? Estimate them.